Risk Propagation in Supply Chains

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The supply chain literature has devoted much attention to studying how the variability of orders propagates upstream. In this paper, instead, we focus on how the variability of payments to suppliers propagates upstream, which has a major impact on risk. To do so, we build a supply chain model based on empirical findings from the financial literature. We show that payments variability may occur even if orders are constant, and that this variability may propagate and increase at upper echelons. We show that this phenomenon is caused by the limited access to debt and identify the factors that drive the propagation of variability—the existence of a financial leverage target, the cost of debt, the firm's operational leverage, and the industry risk. By studying the limiting distribution of the corresponding Markov chain, we numerically illustrate the impact of these drivers on the risk of upper echelons as well as the interactions between order and payment variability. We provide a number of insights and propose measures for risk management.

Key words: risk; bullwhip effect; credit contagion; supply chain-finance link

1.   Introduction

Supply Chain Management (SCM) is concerned with three flows—inventory, information, and money (*e.g.*, Mentzer et al. 2001). Traditionally, in the SC literature, much attention has been devoted to inventory and information flows, and not as much to financial flows. This does not mean that this literature has been oblivious to the importance of finance in operational settings. However, cash is usually seen as a "stock" more than a "flow" (consider, for instance, the financial constraints imposed when investing in capacity or inventory) and, since cash is what ultimately constraints the activity of any firm, not paying attention to financial flows may lead companies

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to financial distress and, possibly, bankruptcy. Paying attention to financial flows is even more relevant in the current economic context, when firms are more leveraged[1] and have more difficulties to raise additional funds. In addition to that, supply chains are becoming longer and broader, making it more difficult to assess the overall effect of decisions made on both realms, operational and financial (Manhattan-Associates 2008).

A key observation that motivated this work, and that may well be driven by some of the facts just mentioned, is the existence of the so called "financial contagion," an effect that arises when firms facing customers' defaults on trade credit (*i.e.*, customers paying later than agreed) are more likely to default themselves to their suppliers (Boissay and Gropp 2007). This phenomenon has been addressed in the financial literature (*e.g.,* Allen and Gale 2000, Egloff et al. 2007). The existence of financial contagion suggests not only that payments to suppliers are subject to variability, but that that variability is somehow transmitted upstream. In the particular case of a firm holding inventory, the relationship between payments from customers and payments to suppliers may be intricate. In fact, material and financial flows are intimately related. Acquiring inventory today entails paying suppliers now or in the future, and decisions on how much inventory to buy in a period may well depend on when financial inflows from customers are estimated to occur.

If cash followed the order flow patterns, the variability of payments would exactly replicate the variability of orders and any of the causes of the bullwhip effect (Lee et al. 1997) would create payments variability amplification. However, in reality, cash flows may deviate from order flows, and payment variability may occur even if there is no order variability.

In this paper, we are interested in exploring the causes and conditions of payment variability and amplification beyond the underlying order variability and amplification. Understanding the variability of payments is key to quantifying the risk of a firm, an issue of major importance, as the impact of financial contagion may spread from a single diad to an entire industry, and even the whole economy (Bardos and Stili 2007). The crisis that began in 2007 is a good example of widespread

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[1] For instance, our calculations show that retailers' financial leverage in the US has increased by 40% in the last 40 years. Source: COMPUSTAT, US retailers (SCI Code 5331) 1969-2008.

contagion because of "massive illiquidity" (Tirole 2010). Right after the Lehman Brothers episode in September 2008, the credit crisis worsened among financial institutions precisely because of the fear of financial contagion (Jorion and Zhang 2009).

Three recent trends make financial contagion particularly relevant. First, firms heavily rely on trade credit. During 2001 in France, "trade payables stood at 103% of manufacturing firms' financial debt and 219% of their bank borrowing "(Bardos and Stili 2007). For US retailers, accounts payable represent 53% of long-term debt[2] and have increased by 38% in 40 years with respect to total assets.[3] Second, firms default on trade credit agreements. According to the National Survey of Small Business Finance, as much as "46% of the firms declared that they had made some payments after the due date during the last year" (Cuñat 2007). Similar claims are found in Boissay and Gropp (2007). The consequences of trade credit defaults can be so strong on a supplier that they may push the company to bankruptcy. In fact, bankruptcy is caused by customers' bankruptcy or default on trade credit in 10% to 20% of the cases (Blazy and Colombier 1997). Finally, firms who rely more on trade credit are more likely to go bankrupt themselves. This phenomenon increases the probability of starting or passing the financial "disease"(Boissay and Gropp 2007).

By better understanding the mechanisms that create and propagate payments variability, we hope to shed light on how to improve the ability of firms and regulators to prevent the potential undesired effects of this variability, such as financial distress and bankruptcy.

To address our research questions, we define a simple supply chain where the first echelon faces stationary random demand. Every echelon pays its only supplier following simple rules, which are mainly grounded in the financial empirical literature (Boissay and Gropp 2007, Bardos and Stili 2007). On analyzing this model, we study the limiting distributions of each echelon's payments to its suppliers, observe how much the variability of payments gets propagated for various sets of parameters, and identify the drivers of such a propagation.

[2] Source: COMPUSTAT, mean value for US retailers (SCI Code 5331) 2004-2008.
[3] Source: COMPUSTAT, US retailers (SCI Code 5331) 1969-2008.

To facilitate the analysis, our model is parameterized so that orders are constant for all echelons, *i.e.*, we eliminated the causes of the order bullwhip effect (Lee et al. 1997). Despite this, we still find that payments variability and amplification may occur. We show that these effects are caused by the inability of buyers to unlimitedly access financial debt and modified by the drivers that impact industry, operational, and financial risks. We find that managers' decisions on these drivers impact not only their firms' returns, but also those of its higher echelons in the supply chain. The impact on upstream suppliers may be significant unless at least one of the firms in the supply chain exhibits "solid" financial statements (for instance, low financial leverage), which would mitigate the propagation of payments variability, a result in line with Boissay and Gropp (2007). However, when the echelons in the chain have "weak" financial statements, payment variability spreads and can even be amplified in upper echelons. Finally, when managers consider the firm's financial status to inform their operational decisions, we find that they might increase order variability but decrease overall payment variability. While this has the desired stabilization effect for the immediate echelons, new payment variability, and hence risk, is introduced for upper echelons of the supply chain.

The rest of the paper is organized as follows: §2 relates our work to the extant literature. In §3 we present the model and derive basic propositions about risk propagation in the supply chain. Then, we explore the impact of the firm's industry, operational, and financial risk on payment variability as well as its sensitivity to managerial risk aversion (§4). We discuss managerial and regulatory implications of our work in §5 and conclude placing our findings in the research context in §6.

2. Literature Review

Some of the mechanisms of risk transmission have already been studied. Within the financial literature, the empirical work by Bardos and Stili (2007) summarizes the work in Stili (2003), who studies defaults on trade credit in France and the "risk contagion" phenomenon, *i.e.*, how borrowers' risk is transmitted to lenders. They identify patterns necessary for risk contagion and bankruptcy to occur and find that risk transmission occurs when receivables represent a significant

portion of total assets (Bardos and Stili 2007). Interestingly, they state that payment defaults are mainly provoked by retailers and wholesalers (43%), and most likely absorbed by wholesalers (80%). Boissay and Gropp (2007) extend the latter work and focus on the propagation of risk in long chains. They argue that trade credit default chains exist, and that firms that have difficulties to access new funds pass the liquidity shocks they face to their suppliers. They identify the existence of "deep pockets", *i.e.*, firms with robust balance sheets who stop the chain of defaults by not passing the liquidity shocks to the suppliers, and who inject liquidity in the industry, which is allocated where is needed the most. They also state that, even when firms have suffered trade credit defaults, they continue to give trade credit, providing some sort of insurance to their customers.

This does not mean that firms continuously default on their suppliers. On the contrary, according to Cuñat (2007), trade credit is used only when other forms of credit (debt holders, shareholders) are not available. Cuñat (2007) presents a theory on the role of trade credit agreements. In this paper, we do not look at trade credit agreements *per se*, but rather assume them and look at trade credit defaults taking these agreements as given.

The financial literature also provides models on credit contagion. Kiyotaki and Moore (1997) define a network of firms to study how shocks propagate and why firms do not insure against accounts receivable shocks. They also study the relation between credit limits and collateral prices, and find that this relation plays a key role in the transmission of shocks. Kiyotaki and Moore (2002) study why contagion seems to be country-dependent. More recent financial models resort to using Markov chains (Giesecke and Weber 2006, Frey and Backhaus 2004, Egloff et al. 2007). All these financial models are high-level, parsimonious models, where the local interactions between firms are not always entirely captured. For example, they tend to ignore the effect of inventory decisions on credit chains. This is rather surprising since a large proportion of trade credit defaults occurs among wholesalers and retailers (Bardos and Stili 2007), who hold large levels of inventory (Gaur et al. 2005), and usually have the ability to decide on their inventory level target.

In the operations management literature, the models on risk propagation are more specific about the local interaction between firms. Battiston et al. (2007) study bankruptcy propagation (up and

downstream) in production networks connected by credit ties, where firms have to adapt when one of the firms in the network goes bankrupt. Using simulation, they measure the impact of the cost of debt and the ability to adapt on production levels and growth. Tsai (2008) assesses the impact of reducing the cash-to-cash cycle on the risk of firms, in a setting where early-payment discounts are offered due to the uncertainty about the time when receivables are paid. The variability of cash flows is specifically used to measure risk. Xiaoyan et al. (2010) propose how to use collaborative formulas between firms, such as information sharing and vendor-managed inventory, to reduce the probability of bankruptcy.

Our model borrows the findings from the empirical financial literature mentioned above to state its assumptions, but differs from the existing models in several ways. Firstly, it describes the interactions between the echelons in the supply chain in more detail, considering the impact on the different elements of the balance sheets of the firms. Secondly, it does not focus on the effects of variability (*e.g.*, bankruptcy), but on the mechanisms that drive such effects. Thirdly, it considers the role of inventory decisions that managers make in response to the financial or operational status of the firms they manage, and measures the impact of such inventory decisions on the risk propagation. Finally, it specifically explains how the payments variability gets created, propagated, and amplified as one moves upstream of the supply chain.

3. Framework

As noted, our main concern is to study the variability of received payments and its upstream propagation. This variability has close ties with firms' risk. The CAPM (Sharpe 1964) asserts that the covariance of the firm returns with the market returns is the key driver of risk when assets can easily be traded and shareholders hold well-diversified portfolios. These assumptions may not hold true in practice, specifically investors may not hold such efficient portfolios. If so, other measures, as the variance of the firm's returns, may be appropriate (Van Mieghem 2007). This variance may well be highly correlated with the variability of payments received (Pérez-González 2003), and, then, the variability of payments received may be a good approximation for shareholders' risk when

there is no diversification (Tsai 2008). By choosing the variability of payments received to measure risk, we ignore the impact of the financial and operational leverages of the firm on its own risk, but still we consider the impact of those on the risk of upper echelons, since the payments made to suppliers depend on the own financial and operational leverages.

In the next sub-section we describe our model in which the key financial assumptions—such as those related to capital structure, dividends policy, or payment priorities—are grounded on findings from the empirical financial literature. We provide the relevant references to the empirical literature as we present our model.

3.1. Model

Consider a serial supply chain consisting of a number of echelons, of which we focus on the first $n+1$: a retailer ($i=1$) and its subsequent suppliers ($i=2,3,\ldots,n+1$), where each echelon is served by only one supplier (see Figure 1). The retailer is a newsvendor who faces i.i.d. demand, ξ_1, and sells at constant price, k_1. Eventual leftovers at the end of each period may be kept or lost. When computing her order quantity, q_1, the retailer is oblivious to the financial situation of the firm, and so she always orders a quantity to maximize her expected profit—this assumption is relaxed in §4.4. We assume a zero lead-time for upper echelons, thus the i-th echelon buys the demand he faces, $\xi_i = q_{i-1}$, and sells it at constant price, k_i, $i=2,3,\ldots,n$. Unless otherwise stated, i takes herein values $1,2,\ldots,n$.

Figure 1   Notation for demand, orders, payments, costs, and price.
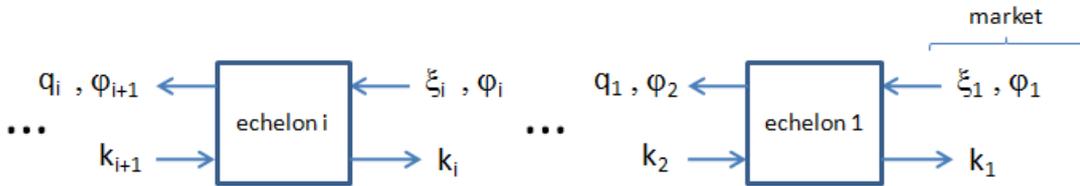
All echelons have access to financial debt and trade credit as sources of funds. Financial leverage is represented by the parameter θ_i, defined as the ratio of financial debt, d_i, and equity level at book value, e_i. Financial debt includes cash, so it can be negative. Contracts may be set such that buyers pay either cash or on credit to suppliers. As in Cuñat (2007), sellers accept that buyers

default on trade credit up to a certain limit, θ_i^b, *i.e.*, as long as $\theta_i \leq \theta_i^b$, and all echelons define a leverage target, θ_i^*. If leverage falls below that target, excess cash is paid to the shareholders as dividends (*e.g.*, Lin and Drekic 2003). The financial debt provider (*e.g.*, a bank) imposes a limit, θ_i^f, on the financial leverage of firm i, above which additional financial debt cannot be obtained. We assume herein that $\theta_i^* \leq \theta_i^f \leq \theta_i^b$. The cost of debt and equity level at each echelon, r_i and e_i, are constant and exogenous.

The order of events is as follows

1. The retailer orders and receives q_1 units from her supplier.

2. Demand is realized and the corresponding cash is collected by the retailer. Leftovers are either kept or lost. Unsatisfied demand is lost.

3. We assume that there is some "pecking order" when using trade credit, since firms may resort to trade credit only when other forms or credit are not available (Cuñat 2007). Thus, the retailer's payments are made according to the following list of decreasing priorities

    (a) Salaries, general and interest expenses. Fixed costs and interest expenses are paid first.

    (b) Supplier. If there is not enough cash to pay the supplier, debt is raised from a bank to complete the payment, up to a limit, θ_1^f. Only if the payment cannot be completely satisfied yet, the buyer defaults on trade credit, accounts payable are increased, and the retailer still has an obligation to pay for the unsatisfied portion the following period (Boissay and Gropp 2007). Partial payments are accepted. If accounts payable grow beyond a threshold, such that $\theta_1 > \theta_1^b$, the firm cannot resort to additional sources of funds and goes bankrupt. In this case, the portion of payables that make leverage exceed the limit θ_1^b are released.

    (c) Bank. If possible, a portion of the principal of the debt is repaid so as to reduce the financial leverage of the firm down to the leverage target, θ_1^*.

    (d) Shareholders. All remaining cash, if any (*i.e.*, if there is still cash available after reducing leverage down to the target, θ_1^*), is paid as dividends.

4. Supplier i $(i = 2, 3, \ldots, n)$ receives its immediate buyer's payment (φ_i) and immediately pays its own supplier, following a list of priorities as in point 3 above.

To write the implied dynamics mathematically, let a second subscript denote time (*e.g.*, $\varphi_{1,1}$ is the payment received by the retailer at $t = 1$), with $t = 0, 1, \ldots$

Since ξ_1 is i.i.d. and leftovers are lost, the retailer always orders the same amount—this assumption is relaxed in §4.4. The order quantity is defined as

$$q_{1,t} = \arg\max_{q_{1,t} \geq 0} \{k_1 \mathbf{E} \min(\xi_{1,t}, q_{1,t}) - k_2 q_{1,t}\} \tag{1}$$

For upper echelons, however

$$q_{i,t} = \xi_{i,t} = q_{i-1,t}, \;\; i = 2, 3, \ldots \tag{2}$$

Let m_i be the market payment less fixed costs and interest expenses. For tractability, demand is set such that m_i is always non-negative

$$m_{1,t} = k_1 \min(\xi_{1,t}, q_{1,t}) - f_1 - r_1 d_{1,t-1} \tag{3}$$

Similarly, for upper echelons, m_i is the buyer's payment less fixed costs and interest expenses

$$m_{i,t} = \varphi_{i,t} - f_i - r_i d_{i,t-1}, \;\; i = 2, 3, \ldots \tag{4}$$

The money available to pay the seller, l_i, depends on the current disposable amount of financial debt plus the period's buyer's payment surplus, $m_{i,t}$

$$l_{i,t} = \theta_i^f e_i - d_{i,t-1} + m_{i,t} \tag{5}$$

While the market pays cash on time (*i.e.*, $\varphi_{1,t} = k_1 \min(q_{1,t}, \xi_{1,t})$), the actual payment to sellers is the minimum of two quantities: the money available to pay and the money owed to the seller, which is the sum of this period's purchasing cost plus the previous accounts payable ($y_{i,t-1}$), if any

$$\varphi_{i+1,t} = \min(l_{i,t}, k_{i+1} q_{i,t} + y_{i,t-1}) \tag{6}$$

Shareholders dividends, $v_{i,t}$, will be paid only if there is money available after paying the supplier and repaying the principal of the debt such that the leverage target is reached

$$v_{i,t} = \max(m_{i,t} - \varphi_{i+1,t} - (d_{i,t-1} - \theta_i^* e_i), 0) \tag{7}$$

The financial debt is limited by the bank's imposed limit, above which the firm cannot resort to additional funds from banks.

$$d_{i,t} = \min(\theta_i^f e_i, d_{i,t-1} + \varphi_{i+1,t} - m_{i,t} + v_{i,t}) \tag{8}$$

Note that the financial debt may be negative if the leverage target is negative.

To calculate accounts payable, if the firm has gone bankrupt in the current period, some payables are released such that the firm is left on the edge of bankruptcy; otherwise, payables increase by the unsatisfied portion of the corresponding current period's payment.

$$y_{i,t} = \min(y_{i,t-1} + k_{i+1}q_{i,t} - \varphi_{i+1,t}, (\theta_i^b - \theta_i^f)e_i) \tag{9}$$

Finally, the buyer's "debt position", x_i, is defined as the sum of financial debt plus accounts payable

$$x_{i,t} = d_{i,t} + y_{i,t} \tag{10}$$

In the next subsection we use this simple model to explore the risk creation and propagation through the supply chain.

3.2. Additional payment variability creation

In this simple model, if firms had unlimited access to financial debt, then buyers would always be able to pay sellers on time and payments would follow the same pattern as orders and inventory (lagged if lead-time were not zero or trade credit were allowed), and so the variability of payments would exactly replicate the variability of orders. However, if the access to financial debt is limited, firms may not be able to pay sellers on time, thus increasing variability of payments as some times the payment will be lower than the corresponding order and, later, the payment will be higher than the corresponding order to reduce the buyers' payables. Therefore, it should be apparent that the limited access to financial debt may create variability of payments beyond the variability of orders.

Consider any diad buyer-seller. Let ξ_1 be an i.i.d. discrete demand distribution with positive support. Assume that borrowing money from a bank is partially restricted for the buyer ($\theta_i^f < \infty$), the seller never triggers the bankruptcy procedure ($\theta_i^b = \infty$), the limiting distribution of payments

to the seller, φ_{i+1}, exists, and payments are made τ periods after orders are placed, with $\tau > 0$. Partial payments are not allowed.

PROPOSITION 1. *Under the proposed scenario, if the buyer does not pay the seller on time with positive probability, then the variability of the payments to the seller is larger than the variability of the orders placed to the seller. Mathematically,* $Prob(l_{i,t} < k_{i+1} q_{i,t-\tau}) > 0 \Rightarrow \boldsymbol{CV}\varphi_{i+1} > \boldsymbol{CV}\xi_{i+1}.$

(*Proofs are shown in the appendix*)

Note that we assess order and payment variability using the coefficient of variation. This allows us to compare variability across distributions with different means, *e.g.*, $\mathbf{E}\varphi_i = k_i \mathbf{E}\xi_i$.

The condition of the proposition may hold in practice, for instance, if there is a number of periods with relatively low demand during which the buyer should face her obligations to pay the seller, but cannot resort to a bank to raise additional funds. However, such a condition never holds if the access to financial debt is not restricted, since the bank could always provide the buyer with the funds necessary to pay the seller on time. If financial debt were not restricted, the variability of payments and orders would be identical. Only if financial debt is restricted, the variability of payments may be greater than the variability of orders, *i.e.*, some additional variability is created. The difference between coefficients of variation of payments and orders represent this additional variability created.

We now show that, once introduced into the supply chain, the variability of payments propagates through it.

3.3. Payment variability propagation

If created, additional variability may be propagated upstream, since it may have an impact on the sellers' ability to pay their own suppliers on time.

Consider the scenario presented in the previous section, whit $\tau \geq 0$.

PROPOSITION 2. *Under the proposed scenario, if* $Prob(l_i < k_{i+1} q_{i,t-\tau})$ *increases when* $\mathbf{CV}\varphi_i$ *does, then* $\mathbf{CV}\varphi_{i+1}$ *increases as well when* $\mathbf{CV}\varphi_i$ *does.*

Again, note that if the access to financial debt is not restricted, the condition of the proposition never holds, since the seller can always pay on time his supplier, and the variability of the payments to the supplier would not increase with the variability of the payments to the seller, but would be identical to the variability of the orders placed to his supplier. Only if financial debt is restricted, some variability may be transmitted from the seller to his supplier. In the latter case, note that not all variability is transmitted, since the buffer of cash provided by the seller's access to financial debt (plus the period's m_i and the cash reserve, if they are positive) absorbs a portion of that variability.

3.4. Payment variability amplification

If variability is propagated, a relevant question is also whether it increases or not upstream, *i.e.*, whether there is variability amplification *à la* Lee et al. (1997) in the payment space.

Let φ_1 be an i.i.d. distribution, with $\varphi_1 = q_1 B(.5)$, where $B(\cdot)$ is a Bernouilli distribution, $k_1 = ck_2$, with $c > 2$ (to avoid trivial solutions) and integer, and no fixed costs. Assume that borrowing money is not an option ($\theta_1^f = 0$), the seller never triggers the bankruptcy procedure ($\theta_1^b = \infty$), and cash available at the end of the period is paid as dividends ($\theta_1^* = 0$).

PROPOSITION 3. *Under the proposed scenario, the variability of the payments to the seller is larger than the variability of the payments to the buyer,* i.e., $\mathbf{CV}(\varphi_2) > \mathbf{CV}(\varphi_1)$.

Note that the result could apply to any diad buyer-seller if the buyer faced a Bernouilli distribution. Again, the assumption that financial debt is restricted is key for the proposition to hold. Furthermore, the conditions of the proposition ensure that the cash buffers are absent both in the firm (through an aggressive dividend policy) and in banks (since raising financial debt is forbidden). Under these conditions, the variability of payments is amplified as it propagates through the supply chain.

In sum, the three propositions above state that, under reasonable conditions, variability of financial flows may be created, propagated upstream, and amplified if access to the financial markets is partially restricted. These findings resonate with work previously done on order amplification in

supply chains (Sterman 1987, Lee et al. 1997). While Lee et al. find four necessary and sufficient conditions for order amplification, we identify one necessary condition for payment variance and amplification (limited access to debt.) Furthermore, just like order amplification creates operational distress in upstream echelons through higher production costs, payment amplification creates operational distress in upstream echelons though higher financial cost.

Finally, it should be noted that, in the order space, the use of order variance as a measure of comparable variability (Lee et al. 1997) requires the assumption of a single unit of input for unit of output. In our case, because of the different prices among echelons, we use the coefficients of variation of payments as a comparable metric of variability. This adjustment of metric would also be necessary to compare order variability if multiple units of input were required per unit of output, *e.g.*, tires for a vehicle.

In the following section, we explore the effects of different risk sources on payment variability, how they impact its propagation in the supply chain, and how payment variability interacts with order variability.

4. Numerical study

In this section, we first characterize the Markov chain that results from the model defined so as to calculate the payment limiting distributions, and then numerically study the impact of various drivers on variability creation, propagation, and amplification.

### 4.1. Characterization and limiting distributions

Given the structure of equations (1) to (9), we note that the process defined above can be described as a Markov chain, where the state is defined by the debt positions of a buyer and her supplier, *i.e.*, the duplet (x_i, x_{i+1}). For convenience, we will focus on the diad retailer-wholesaler (herein denoted by subscripts r and w), and it will be useful to also consider the wholesaler's supplier, named the manufacturer (m). Note, however, that the model, and consequently the findings, are transferable, *mutatis mutandis*, to all the buyer-seller diads in the serial supply chain.

The state space is limited by two barriers for each echelon, one due to the payment of dividends, another due to the trigger of the banktuptcy procedure.

If the demand distribution is discrete, debt is costless and, at each period, retailer and wholesaler's revenue (expenses) outweighs expenses (revenue) when the demand is the highest (the lowest), then the chain is ergodic, since all states are recurrent. If debt is not costless, the number of states becomes infinite.

Figure 2 illustrates the behavior of the Markov chain. It consists of two special random walks, one for the retailer, another for the wholesaler, where the magnitude of the change in the debt position depends on the state and the parameters. The two random walks are linked because the magnitude of the wholesaler's change in the debt position depends on the retailer's state. For each echelon, there are two barriers that limit the state space, defined, respectively, by the parameters θ_i^* (leverage target barrier) and θ_i^b (bankruptcy barrier), $i \in \{r, w\}$.

Figure 2 **Pictorial description of the random walks. The magnitude of the change in the debt position changes with the parameters of the model and the state.**
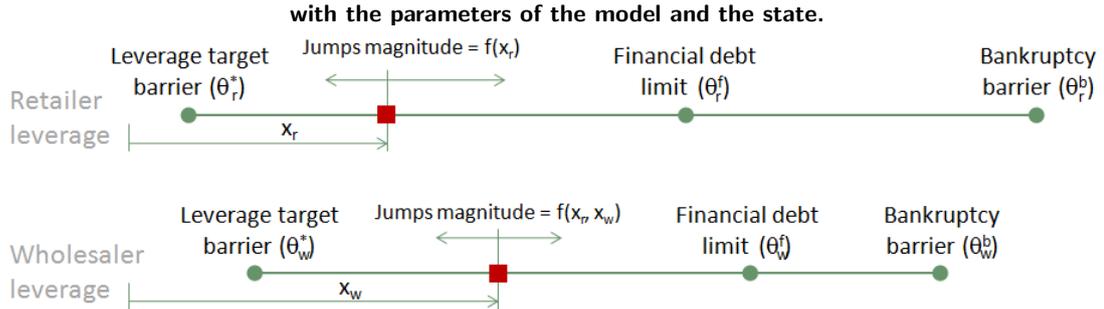
Our concern is to find how the limiting distribution of payments made by the retailer (wholesaler) is related to that of the market (retailer) depending on the parameters. In particular, we focus on the variability of the payment distributions, as they are related to the risk faced at each echelon. We find the payment distributions for both retailer and wholesaler through the estimation of the limiting distribution of the Markov chain, whose transition-probability matrix, P, is derived according to equations (1) to (9). Unless otherwise stated, we limit ourselves to settings where the chain is finite, so that P is also finite.

Denoting by π the column vector of probabilities for the limiting distribution, π satisfies $\pi = P\pi$, with $\sum_{i=1}^{N} \pi_i = 1$, where N is the number of states. Let $\hat{P}$ be the transpose of the matrix that results

from substituting the last row of $P - I_N$ by ones, where I_N is the N-dimension diagonal matrix. It can be shown that $\pi = \hat{P}^{-1}e_N$, where e_N is the N-dimension column vector $(0, 0, \ldots, 0, 1)$.

To find the distribution of payments, note that the payments to the sellers, φ_w and φ_m, are unambiguously defined by the state and the demand realization. Given π, the limiting distribution of the payments, φ, can be calculated as

$$\varphi = C\pi = C\hat{P}^{-1}e_N \tag{11}$$

where C is an $M \times N$ matrix, whose elements c_{ij} contain the probability of payments $(\varphi_{wi}, \varphi_{mi})$ given the state (x_{rj}, x_{wj}), with M being the cardinality of the state space of payments.

4.2. Impact of drivers on payment variability

Equipped with the expression for the payment limiting distributions, we are ready to study the impact of different drivers on the distributions. According to Propositions 1 and 2, the amount of money available to pay the seller, l, is the variable that may trigger variability creation and propagation. In turn, l depends on the model parameters. In this section, we observe payment variability as some of these parameters change, either in isolation or simultaneously.

Unless otherwise noted, the following set of parameters is used.

Parameter	Value(s)
Retailer's price	$k_r = \$10$
Variable cost	$k_w = \$4, k_m = \$2$
Fixed cost	$f_r = \$300, f_w = \$150$
Equity level	$e_r = e_w = \$1,000$
Leverage target (desired financial debt/equity)	$\theta_r^* = \theta_w^* = 0.3$
Financial limit (max. financial debt/equity)	$\theta_r^f = \theta_w^f = 0.6$
Bankruptcy limit (max. debt position/equity)	$\theta_r^b = \theta_w^b = 0.9$
Cost of debt	$r_r = r_w = 0$

A fortiori, we assume that the demand observed by the retailer can take two values, 50 and 100, with probability $1/2$ each. These values ensure that the resulting Markov chain is ergodic, the state space finite, and m_r non-negative. More involved demand distributions increase computational time but do not change the qualitative results (one example is shown in §4.4.) For convenience, we also assume that trade credit is not used, although this assumption can be easily relaxed, and we

assume, without loss of generality, that $x_{r,0} = x_{w,0} = 0$. Finally, we assume that leftovers are lost, so retailers and wholesaler's orders are constant over time.

Given these parameters, the retailer will order 100 units every period (*i.e.*, $\mathbf{CV}\xi_w = 0$), and will have problems to honor her payments to the wholesaler when her debt position is larger than \$600, creating variability on the payments the wholesaler receives. This set of assumptions eliminate all the causes of the bullwhip effect in orders (which would create, propagate, and amplify payment variability) and allows us to perform an incremental analysis on the drivers of the retailers payment variability. Note that if the financial limit for the retailer were infinite, the payment variability would always be equal to the order variability, *i.e.*, zero.

As pointed out in the literature —*e.g.*, Penza and Bansal (2001)—the drivers that modify firms' risk may fall into the following three main categories of risk: industry, operational, and financial. We explore first the industry and operational risks, and then the financial risk.

4.2.1. Industry and operational risks We first explore the impact of the industry risk and the operational leverage of the retailer on the variability of payments to the wholesaler. Industry risk depends on the volatility of the industry demand and can be measured by demand variability, whilst operational risk depends on the technology chosen by the firm and can be measured by the proportion of fixed costs with respect to total costs.

Figure 3 shows the coefficient of variation of the payments received by the wholesaler as a function of the demand coefficient of variation and the retailer's level of fixed costs. Analysis of these results reveal some interesting observations. First, since $\mathbf{CV}\xi_w = 0$, if the access to financial debt were not limited, variability would be zero. Therefore, all the variability in Figure 3 has been created due to lack of access to financial debt, a fact in line with Proposition 1. Second, the payment variability increases with the demand variability. The higher the demand variability, the higher the probability that the retailer cannot pay the wholesaler on time, what increases the payment variability to the wholesaler according to Proposition 2. Interestingly, if price increases, *ceteris paribus*, the demand variance increases but the variability of payments decreases. The reason is that, as price increases,

Figure 3   Payment variability as a function of industry and operational risks.
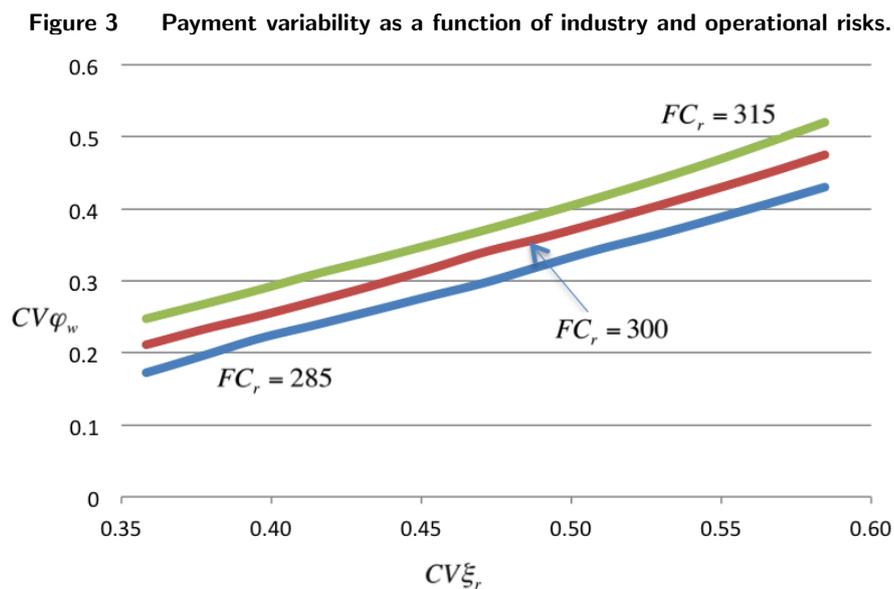
more money is injected on average into the firm, which acts as a cash buffer that reduces the variability of payments. Third, payment variability to the wholesaler increases with the level of fixed costs, a measure of operational leverage. The higher the retailer's fixed costs, the average money available to pay the wholesaler decreases but the variance of payments remains the same, and thus the coefficient of variation increases.

Although all the points mentioned above are considered from the retailer's point of view, the most relevant fact about them is that they have an impact on the supplier's risk. For instance, the type of technology chosen by the retailer has an impact on the supplier, who sees that his risk profile may worsen due to an exogenous decision to him.

4.2.2.   Financial risk We now explore the impact of financial risk on the variability of the payments to the wholesaler. Financial risk depends both on the firm's capital structure and the cost of debt. Figure 4 shows the payments coefficient of variation as a function of the retailer's leverage target and cost of debt.

Because, in this case, we allowed $r_i > 0$ and the number of states is infinite, it was not possible to find the limiting distribution as per equation 10. Instead, we relied in the observed distribution of a simulation with 10,000,000 periods captured after 1,000,000 periods of simulation to eliminate

possible initialization transients—see Law and Kelton (2000, section 9.5.1) for a discussion on the warm-up periods for simulation models of limiting distributions. This simulation horizon and warm-up periods were enough to ensure that simulation errors of the output were almost always below 0.1% for the extensive tests we made using both simulation and Equation 11.

Figure 4   Payment variability as a function of leverage target and cost of debt.
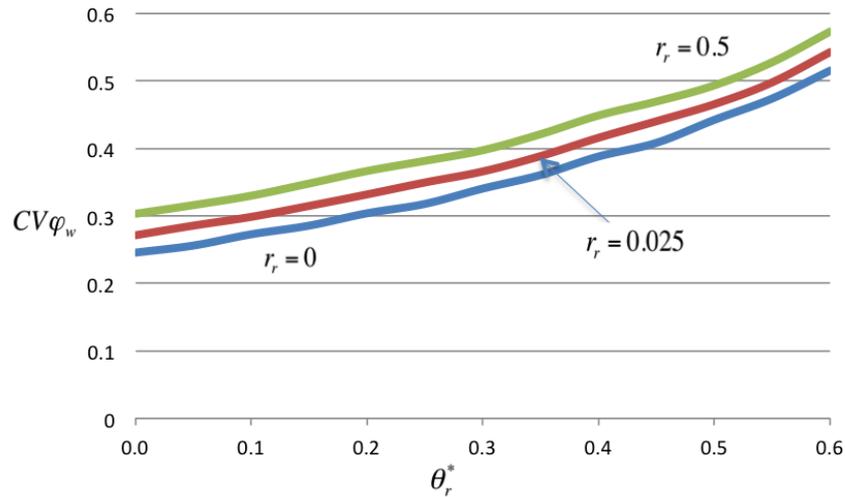
Note that the payment variability increases with the leverage target. When this target increases, the dividends barrier is moved to the right in Figure 2 and dividends are paid more frequently, diminishing in more periods the buffer of cash available to pay the supplier, hence increasing the probability of both not paying on time and paying larger quantities in subsequent periods, which increases variability. A key observation is that the existing cash buffers (*i.e.*, the cash on hand plus the additional financial debt available) act in a similar fashion as inventory buffers in inventory models. Cash buffers may attenuate the payment variability in the same way as inventory buffers may attenuate the order variability. Note also that the cost of debt creates additional payments variability, although the underlying mechanism is different from the previous example. If the cost of debt increases, the retailer's debt position is pushed to the right in Figure 2, and the retailer has more difficulties to pay the wholesaler on time, again increasing the probability of paying larger quantities in subsequent periods, hence increasing variability. This mechanism is attenuated by the payment release in case of bankruptcy, since subsequent payments would be larger if some

payments were not released. Finally, if the cost of debt were an increasing function of the debt position, as it is usually the case in reality, the variability would grow even more. Numerical tests not reported here confirm this intuition.

Note that payment variability increases with the usual measures of risks (*e.g.*, leverage target level of fixed costs, demand variability). While this is not surprising, our model allows us to explain the mechanisms through which these risk factors impact the payment variability. Furthermore, these results corroborate our initial hypothesis of payment variability being an appropriate surrogate of risk.

4.3. Variability Propagation

We next explore the impact of one of the risk drivers, specifically, the capital structure, on the propagation of risk through the supply chain. While the specific response to the changes in other drivers might differ from the results reported there, the analysis of one driver allows us to develop qualitative statements on the sign and intensity of these responses. Figure 5 shows the impact of the retailer's leverage target on the payments received by both the wholesaler and the manufacturer. As the retailer's financial leverage increases, not only the variability created at the retailer is passed

Figure 5   Payment variability as a function of the retailer's leverage target.
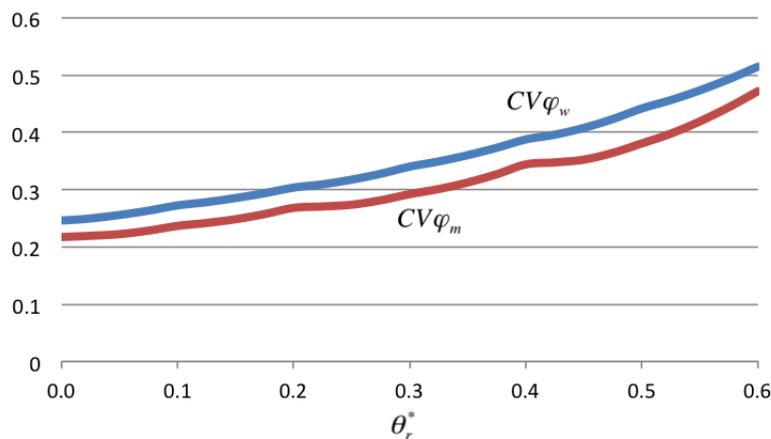

to the wholesaler, but a significant portion of that variability is in turn passed to the manufacturer. The same propagation mechanism may be replicated for higher echelons in the supply chain, and

also for other drivers. In fact, the propagation phenomenon just described may well lead to the appearance of credit chains and financial contagion, in line with the observations in the financial literature (*e.g.,* Boissay and Gropp 2007).

To explore the propagation of payment variability upstream, we fix the retailer's leverage target at 0.3, and let the wholesaler's leverage target vary from -0.6 to 0.6. Figure 6 reports the ratio of $\mathbf{CV}\varphi_m$ to $\mathbf{CV}\varphi_w$ as the wholesaler's leverage target varies. Observe first that variability is greatly attenuated if the wholesaler's leverage target is low. The wholesaler is injecting liquidity to the

Figure 6 **Variability amplification factor as a function of the wholesaler's leverage target.**
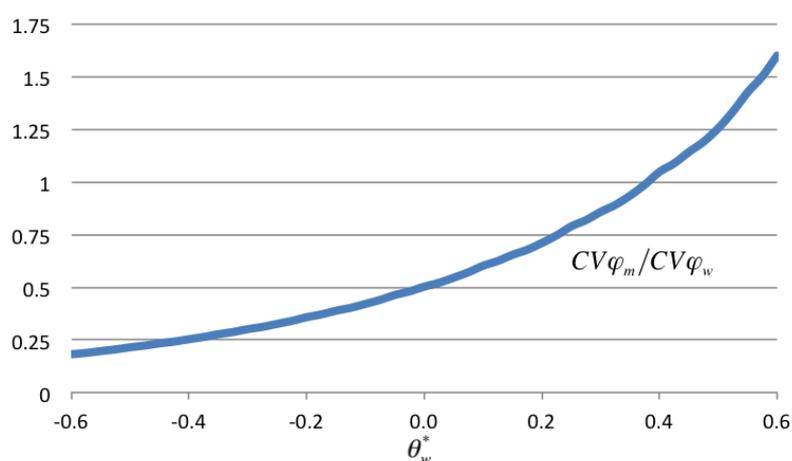
supply chain, and does it where is needed the most, a result in line with Boissay and Gropp (2007). Thus, in terms of financial contagion, the supply chain is de-coupled by the presence of firms with deep pockets in the supply chain. Note also that variability is propagated when the wholesaler's leverage target is relatively large, since he cannot access to additional funds to buffer against the payment variability he faces. Finally observe that, if his leverage target is above approximately 0.4, then variability is not only propagated, but also amplified, an issue we now turn to.

Figure 7 shows the payment variability amplification measured as the ratio of the coefficients of variation of payments received by the manufacturer and the wholesaler, when the retailer and the wholesaler simultaneously change their leverage targets within the range 0 - 0.6.

Note that for some combinations of leverage targets, variability gets amplified. The mechanism at work is the same as the one explained in the first example in section §4.2.1, but the outcome

Figure 7   Payment variability amplification factor as a function of the retailer and wholesaler's leverage targets.
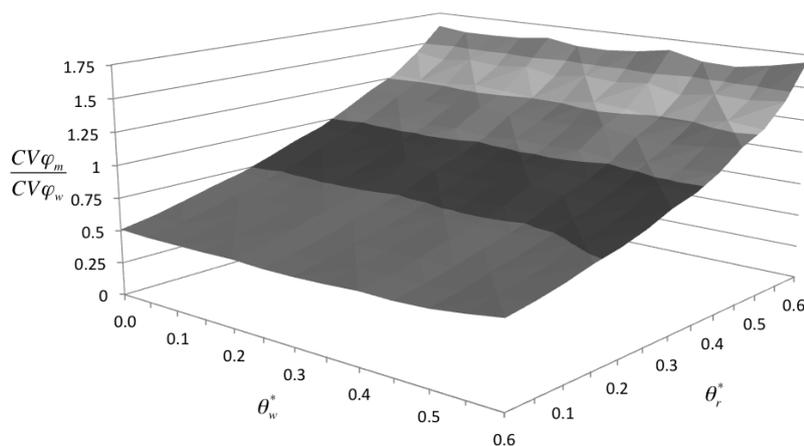
is quantitatively different. In fact, when comparing Figures 5 (when only the retailer increases her leverage target) and 7 (where both players increase their leverage targets), it becomes apparent that, when two consecutive echelons increase their leverage targets simultaneously, the consequences are more harmful for the entire supply chain. This reasoning can be extended to other combinations of drivers, various players in the supply chain, and even various supply chains in the economy. The crisis that started in 2007 may be an example of the consequences of "massive iliquidity" (Tirole 2010) (partially perhaps caused by high leverage targets) at a global scale. Also note that, for a given wholesaler's leverage target, when the retailer's leverage target increases, the amplification factor does not grow monotonically. This does not contradict the fact that the coefficient of variation of payments to the manufacturer has been reported to monotonically increase with the retailer's leverage target (Figure 5).

In this section, we have analyzed the impact of several drivers on the creation and propagation of payment variability. The four drivers identified (industry risk, leverage target, cost of debt, and operational risk) may be seen as the counterparts of lead-time in the order space (Lee et al. 1997), in the sense that all these drivers do no create variability *per se*, but, once variability exists, they aggravate or attenuate its effects.

4.4. Variability interactions

So far, we have assumed that all the causes of the bullwhip effect for orders were absent. Therefore, only the "financial" drivers were accountable for the increase in payment variability. In this subsection, we relax this assumption to observe the interactions between order and payment variability. Specifically, we explore how the financial status of a firm impacts the order behavior, by making the retailer's manager sensitive to the financial situation of his firm and reduce his orders whenever the firm is close to bankruptcy, thus reducing his exposure to bankruptcy distress. We find this driver relevant for two reasons: first, it is a phenomenon that may appear only if the access to financial debt is limited; second, since it impacts the order variability, it has consequences in both spaces, information and financial.

To define a plausible scenario, we consider that the retailer's manager has bankruptcy aversion level a and usually buys the profit-maximizing quantity according to the newsvendor solution in Equation 1 (100 units), but buys $100 - a$ units in those periods where financial leverage is larger than the financial limit, θ_r^f. Figure 8 shows the coefficient of variation of payments to both the wholesaler and the manufacturer, and the coefficient of variation orders received by the wholesaler as a function of the bankruptcy aversion level.

Figure 8   **Payment variability as a function of the degree of aversion to bankruptcy by the retailer's manager**
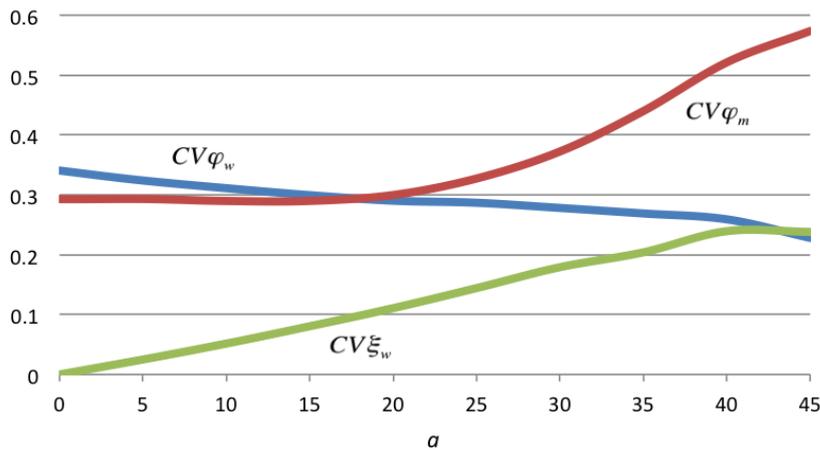
The order variability increases with the bankruptcy aversion level, as the orders placed further depart from the profit-maximizing order quantity. The payment variability faced by the wholesaler

monotonically decreases even if order variability (which, other things being equal, creates payment variability) increases and financial debt is restricted. The retailer's behavior is beneficial for the wholesaler, who faces lower risk as a increases. Interestingly, this benefit is not transferred to the manufacturer. For low values of a, payment variability decreases, but for high values of a, the impact of order variability created by the retailer's behavior overweighs the smoother payment function due to the firm moving away from bankruptcy. The reason is that the payment function is filtered by the wholesaler, but the order function is not. The consequence is that a conservative response in the retailer side does have the desired effect on her performance, but additional risk and variance gets introduced into the upper echelons. These observations should open the door to broader decision models that transcend the operational realm and give opportunity to study more complex interactions between inventory, cash reserves, orders, and cash flows (*e.g.,* Yang and Birge 2011).

5. Managerial Implications

A major concern for managers and regulators may not be how variability is propagated *per se*, but the fact that, as a consequence of that variability, firms in their supply chain may have a higher risk exposure. This is specially important for those companies whose performance strongly depend on their suppliers' survival, such as the automotive industry. Major car manufacturers have programs to assess the probability of first-tier suppliers going bankrupt. In turbulent times, bankruptcies arise and suppliers have to be closely overseen. As an example, consider the crisis that started in 2007. During 2008, roughly 150 (of 1200) of a major German OEM's first-tier suppliers went bankrupt. Having the ability to anticipate suppliers' bankruptcies is key to avoid major disruptions in the channel. To do so, buyers deploy their control programs resorting to available tools, such as Altman's z-score (Altman 1968). A weak point of these tools is that they rely on accounting and market data to compute control indexes, but are oblivious to the mechanisms that drive such data.

Our work may help managers to define additional or alternative control mechanisms to anticipate bankruptcies. A key observation is that, for either echelon, the probability of a firm going bankrupt

depends on the relative size of the magnitude of the change in the debt position with respect to the distance between the two barriers (see Figure 2). For instance, if the variability of the buyer's payments increases, *ceteris paribus*, then the average magnitude of the changes in the seller's debt position increases, and also the probability of the seller going bankrupt. Therefore, as a heuristic, any driver change that increases the relative size of the magnitude of the change in the debt position with respect to the distance between the barriers will increase risk and the probability of going bankrupt, and viceversa. For instance, defining a more aggressive capital structure, by increasing the leverage target of the firm, moves the dividends barrier to the right in Figure 2, shortening the distance between the two barriers, and increasing the probability of the firm hitting the bankruptcy barrier. Likewise, all the drivers we identified have either a direct impact on the magnitude of the change in the debt position (this is the case of cost of debt, operational leverage, and industry risk) or the distance between barriers (leverage target). Indirectly, all of them have an impact on both. For instance, the leverage target also impacts the magnitude of the changes in the debt position through interest expenses.

This heuristic can be formalized by analyzing the behavior of the following ratio:

$$STBI = \frac{\text{Prob}(\Delta x_i > 0)\mathbf{E}(\Delta x_i | \Delta x_i > 0)}{(\theta_i^b - \theta_i)e_i}, \text{ where } \Delta x_i = x_{i,t+1} - x_{i,t}$$

which we name the Short-Term Bankruptcy Index (STBI.) The numerator is the expected change in the debt position to the right (Figure 2) conditional on moving to the right times the probability of moving to the right, and the denominator includes the distance from the current position to the bankruptcy barrier. For instance, for high leverage targets, odds are higher that the state moves to the right due to interest expenses (which may be aggravated by the cost of debt increasing with the state) and the state is closer to the bankruptcy barrier, plus the distance between barriers is lower. Overall, large leverage targets make it more difficult to stay away from the vicinity of bankruptcy. This index takes into consideration the current state of the firm (its debt position), and so it contains information on how close we are to hit the bankruptcy barrier, which may be useful to making tactical decisions. Note that the response of the risk averse manager presented

in §4.4 is affecting this short term ratio as the reduction of orders reduces the magnitude of the change in the debt position once it approaches the bankruptcy limit.

An alternative heuristic would be to ignore the current state, x_i, and just focus on the Long Term Bankruptcy Index (LTBI) by monitoring the ratio of the standard deviation of the retailer's debt position change to the distance between barriers. Formally,

$$LTBI = \frac{\sigma(\Delta x_i)}{(\theta_i^b - \theta_i^*)e_i}$$

By and large, these indexes may be useful to help managers assess the probability of firms (suppliers, customers) going bankrupt and categorize them when devising risk assessment programs, consider alternative suppliers to distressed ones, or select suppliers for new parts. A good feature of these indexes is that all sources of risk (industry, operational, financial) are jointly considered in one measure. A model like the one presented can be used to estimate the factors in the numerators; the remaining information to calculate the indexes may be obtained from the firms' financial statements and through due diligences.

These heuristics may help managers judge how their decisions will impact bankruptcies in the supply chain. For instance, trade credit is sometimes seen as "free lunch" for buyers having a dominant position in the market, such as large retailers or manufacturers. These seem to be able to obtain agreements to pay their sellers later without expecting further negative consequences, such as subsequent price increases, lower quality, or late deliveries. However, other things being equal, reaching agreements to paying sellers later will move the buyer's financial barrier (and hence the bankruptcy barrier) to the left, shortening the distance between barriers, therefore taking both buyer and seller closer to bankruptcy, and increasing *de facto* the risk of both players. When used extensively by all of the large players in the channel, trade credit (typically combined with high financial leverage) may set the conditions for a major disruption in payments, if one of the lower echelons triggers the financial contagion.

The same logic can be applied to other strategies, such as reverse factoring, typically used by strong buyers to get around the financial burden imposed to weak sellers that have to finance

their trade credit agreements. Buyers claim that this is a win-win solution, since they benefit from paying later the sellers, the sellers get better conditions from the lenders, and the lenders increase their income plus have access to new customers. While this may well be true regarding cash flows, it is not clear that value increases for the chain as a whole. The reason is, again, that paying sellers later decreases the distance betweens barriers, rising buyer's cost of capital which also increases the seller's cost of capital. As a consequence, rational lenders should ask for higher returns.

On the other hand, the fact that risk may propagate implies paying attention to whatever occurs up and downstream. For instance, in a B2B environment, managers should be especially cautious when selecting customers, who may bring along an array of supply chain echelons subject to various, and maybe "contagious", levels of risk. Likewise, investors, when deciding on the weight of the stock of a firm in their portfolios, should not be oblivious of the peculiarities of the various echelons related to that particular firm, since the shareholders' cost of capital is affected by what occurs several echelons down the supply chain. By the same token, banks should calculate risk premia depending not only on the characteristics of a firm, but also on those of its customers' customers.

From a regulatory perspective, policy makers may want to impose limits on trade credit agreements that may decrease variability. By doing so, they not only protect the small (usually weaker) players, but all the echelons in the supply chain, the industry, and, eventually, the whole economy. This might be why some regulators are imposing limits to trade credit in some industries, as was recently decided upon in some European countries (Sersiron and Dany 2008). Finally, financial authorities may have to decide where to inject liquidity (*e.g.*, whom to bail out) if needed, since the risk profiles of the different echelons may well depend on where money is injected. For example, it might be more efficient to inject liquidity into a buyer than into a seller that may go bankrupt due to the inability of that buyer to pay on time.

6. Conclusions

This paper studies how risk propagates upstream. We leverage on the existing financial literature to build a theory (as suggested by Boissay and Gropp 2007) on how firms behave when constrained

by limited access to funds. When this factor is present, we show that payment variability may be created, propagated, and amplified. Using Markov chain tools, we conduct a series of numerical studies to assess the impact of various drivers on the variability of payments. We identify four factors that exacerbate its impact: leverage target, cost of capital, operational leverage, and industry risk. Also we find that managers' behavior to avoid bankruptcy reduces payment variability but increases order variability. While this has the desired stabilizing effect for the immediate echelons, this benefit is not transferred to upper echelons as the new order variability results in greater payment variability. Payment variability is a major driver of firms' risk, hence our findings about variability can be extrapolated to how risk is created, propagated, and amplified.

Our results are consistent with Boissay and Gropp (2007) and Bardos and Stili (2007). In fact, our findings support the existence of credit chains and the beneficial impact of "deep pockets", who reduce the propagation of variability. The phenomenon of propagation implies that the risk of a firm is not only driven by its customers, but also by its customers' customers and so on. Likewise, a firm decision has an impact on its cost of capital and that of its suppliers and suppliers' suppliers. Furthermore, the combined effect of two or more exacerbating simultaneous decisions in two echelons of the chain may greatly impact the risk of the array of the upstream suppliers, thus the health of the whole supply chain and, eventually, the economy. Also, Tsai (2008) finds a trade-off between the size of the cash-to-cash cycle and risk, which is also consistent with our findings of risk increasing with the position of the leverage target.

We acknowledge the limitations of our model in several aspects that can be further explored in subsequent work. First, using more realistic settings, such as assuming that firms may have multiple suppliers and customers, can help reveal additional insights. Second, considering alternative dividends policy, such as paying "sticky" dividends, which are common in some industries (Myers 1984), might have an impact on the way variability propagates. Finally, looking at how risk propagates downstream, due to poor quality or service that may ultimately lead to lower and more variable buyer's returns (*e.g.*, Harrison et al. 2009), may nicely complement our work.

Appendix

Proof of Proposition 1 If leftovers are not lost, all echelons will buy the market demand, ξ_1, at any period. If debt were not restricted, we would have: $\text{Prob}(l_{i,t} < k_{i+1}q_{i,t-\tau}) = 0$, $\mathbf{CV}\varphi_{i+1} = \mathbf{CV}\xi_1 = \mathbf{CV}\xi_{i+1}$, and $\mathbf{E}\varphi_{i+1}^2 = k_{i+1}\mathbf{E}\xi_{i+1}^2$. When debt is restricted, it may be the case that $\text{Prob}(l_{i,t} < k_{i+1}q_{i,t-\tau}) > 0$ and then we shall show that $\mathbf{CV}\varphi_{i+1} > \mathbf{CV}\xi_{i+1}$. Since

$$\mathbf{CV}\varphi_{i+1} = \frac{\sqrt{\mathbf{E}\varphi_{i+1}^2 - \mathbf{E}^2\varphi_{i+1}}}{\mathbf{E}\varphi_{i+1}}$$

and $\mathbf{E}\varphi_{i+1}$ does not change when debt is restricted because the seller never triggers the bankruptcy procedure, it will suffice to show that $\mathbf{E}\varphi_{i+1}^2 > k_{i+1}\mathbf{E}\xi_{i+1}^2$.

Let the limiting distribution φ (subscripts $i+1$ are removed hereafter for simplicity) take the values $\varphi_1, \varphi_2, \ldots, \varphi_N$. If $\text{Prob}(l_{i,t} < k_{i+1}q_{i,t-\tau}) > 0$, then some payments to the seller will be zero with positive probability, since partial payments are not allowed. A deferred payment will have to be satisfied in a subsequent period together with the corresponding payment of the period. Let Δ_j be the incremental probability of not paying φ_j in a period because of lack of funds and Δ_k^j the incremental probability of not paying φ_k in a subsequent period because a payment of $\varphi_k + \varphi_j$ is made in such a period. Therefore, the change in the second moment of the payment function when $\text{Prob}(l_{i,t} < k_{i+1}q_{i,t-\tau}) > 0$ is

$$\mathbf{E}\varphi_{i+1}^2 - k_{i+1}\mathbf{E}\xi_{i+1}^2 = -\sum_{j=1}^{N}|\Delta_j|\varphi_j^2 + \sum_{j=1}^{N}\sum_{k=1}^{N}|\Delta_k^j|(\varphi_j + \varphi_k)^2 - \sum_{j=1}^{N}\sum_{k=1}^{N}|\Delta_k^j|\varphi_k^2$$

with $\sum_{k=1}^{N}\Delta_k^j = \Delta_j$. The first term corresponds to the fewer payments of magnitude φ_j made because of lack of funds. The second term corresponds to the additional subsequent payments of magnitude $\varphi_j + \varphi_k$ that have to be made to clear supplier's debt, and the third term includes the fewer payments of magnitude φ_k due to the payments of the second term.

Working out the right-hand side of the previous equation, it can be shown that

$$\mathbf{E}\varphi_{i+1}^2 - k_{i+1}\mathbf{E}\xi_{i+1}^2 = 2\sum_{j=1}^{N}\sum_{k=1}^{N}|\Delta_k^j|\varphi_j\varphi_k > 0$$

and the result follows.

If leftovers are lost, all echelons will buy the retailer's newsvendor fractile at any period, thus $\mathbf{CV}\varphi_{i+1} = 0$, leading to a particular case of the previous one. $\square$

Proof of Proposition 2 The result directly follows from Proposition 1 when leftovers are not lost. $\square$

Proof of Proposition 3   We first show the result for $c = 3$, and then generalize for any $c > 2$ integer.

To find the coefficient of variation of the limiting distribution of payments to the supplier, we calculate the two first central moments, $\mathbf{E}\varphi_2$ and $\mathbf{E}\varphi_2^2$.

To find the first moment, we find $\mathbf{E}\varphi_{2,t}$ for $t = 1, 2, \ldots$ under the conditions of the proposition.

$$\varphi_{2,1} = \min(k_2 q_1, k_1 \varphi_{1,1})$$

$$\mathbf{E}\varphi_{2,1} = \frac{1}{2}(k_2 q_1 + 0) = \frac{k_2 q_1}{2}$$

$$\varphi_{2,2} = \min(k_2 q_1 + (k_2 q_1 - k_1 \varphi_{1,1})^+, k_1 \varphi_{1,2})$$

$$\mathbf{E}\varphi_{2,2} = \frac{1}{2^2}\left(\min(k_2 q_1, 0) + \min(2k_2 q_1, 3k_2 q_1) + \min(k_2 q_1, 3k_2 q_1)\right) = k_2 q_1 \left(\frac{1}{2} + \frac{1}{4}\right)$$

where $a^+ = max(a, 0)$.

Likewise,

$$\varphi_{2,3} = \min(k_2 q_1 + (k_2 q_1 + (k_2 q_1 - k_1 \varphi_{1,1})^+ - k_1 \varphi_{1,2})^+, k_1 \varphi_{1,3})$$

$$\mathbf{E}\varphi_{2,3} = \frac{1}{2^3}(3k_2 q_1 + k_2 q_1 + 2k_2 q_1 + k_2 q_1) = k_2 q_1 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$$

$$\varphi_{2,4} = \min(k_2 q_1 + (k_2 q_1 + (k_2 q_1 + (k_2 q_1 - k_1 \varphi_{1,1})^+ - k_1 \varphi_{1,2})^+ - k_1 \varphi_{1,3})^+, k_1 \varphi_{1,4})$$

$$\mathbf{E}\varphi_{2,4} = \frac{1}{2^4}(3k_2 q_1 + k_2 q_1 + 2k_2 q_1 + k_2 q_1 + 3k_2 q_1 + k_2 q_1 + 2k_2 q_1 + k_2 q_1) = k_2 q_1 \left(\frac{1}{2} + \frac{1}{4} + \frac{2}{16}\right)$$

$$\varphi_{2,5} = \min(k_2 q_1 + (k_2 q_1 + (k_2 q_1 + (k_2 q_1 + (k_2 q_1 - k_1 \varphi_{1,1})^+ - k_1 \varphi_{1,2})^+ - k_1 \varphi_{1,3})^+ - k_1 \varphi_{1,4})^+, k_1 \varphi_{1,5})$$

$$\mathbf{E}\varphi_{2,5} = \frac{1}{2^5}(3k_2 q_1 + 2k_2 q_1 + \cdots + k_2 q_1) = k_2 q_1 \left(\frac{1}{2} + \left[\frac{1}{2^2} + \frac{1}{2^5}\right] + \frac{1}{2^3}\right)$$

$$\varphi_{2,6} = \min(k_2 q_1 + \cdots + (k_2 q_1 + (k_2 q_1 - k_1 \varphi_{1,1})^+ - k_1 \varphi_{1,2})^+ - \cdots - k_1 \varphi_{1,5})^+, k_1 \varphi_{1,6})$$

$$\mathbf{E}\varphi_{2,6} = \frac{1}{2^6}(3k_2 q_1 + \ldots) = k_2 q_1 \left(\frac{1}{2} + \left[\frac{1}{2^2} + \frac{2}{2^5}\right] + \left[\frac{1}{2^3} + \frac{1}{2^6}\right]\right)$$

From the latter equations, note that

$$\mathbf{E}\varphi_{2,2} - \mathbf{E}\varphi_{2,1} = k_2 q_1 \frac{1}{2^2} = \frac{1}{2^2}\frac{3}{3}\binom{0}{0}$$

$$\mathbf{E}\varphi_{2,3} - \mathbf{E}\varphi_{2,2} = k_2 q_1 \frac{1}{2^3} = \frac{1}{2^3}\frac{3}{3}\binom{1}{1}$$

$$\mathbf{E}\varphi_{2,4} - \mathbf{E}\varphi_{2,3} = 0$$

$$\mathbf{E}\varphi_{2,5} - \mathbf{E}\varphi_{2,4} = k_2 q_1 \frac{1}{2^5} = \frac{1}{2^5}\frac{1}{3}\binom{3}{2}$$

$$\mathbf{E}\varphi_{2,6} - \mathbf{E}\varphi_{2,5} = k_2 q_1 \frac{2}{2^6} = \frac{1}{2^6}\frac{1}{2}\binom{4}{3}$$

Working in this fashion, it can be shown that

$$
\mathbf{E}\varphi_{2,t} - \mathbf{E}\varphi_{2,t-1} =
\begin{cases}
\frac{1}{2^t}\frac{3}{t}\left(\frac{t-2}{\frac{2t}{3}-1}\right) & \text{, if } t = \dot{3} \\[2ex]
\frac{1}{2^t}\frac{3}{2t-1}\left(\frac{t-2}{\frac{2t-4}{3}}\right) & \text{, if } t = \dot{3}-1 \\[2ex]
0 & \text{, if } t = \dot{3}-2
\end{cases}
$$

Let $\bar{\varphi}_{1,t}$ be a vector containing the first t random variables $\varphi_{1,1}, \varphi_{1,2}, \ldots, \varphi_{1,t}$, and $\hat{\varphi}_{1,t}$ a vector of realizations of these random variables. Given these definitions, for any $t = \dot{3}$, we can write

$$
\varphi_{2,t} = \min(k_2 q_1 + \cdots + (k_2 q_1 + (k_2 q_1 - k_1\varphi_{1,1})^+ - k_1\varphi_{1,2})^+ - \cdots - k_1\varphi_{1,t-1})^+, k_1\varphi_{1,t})
$$

$$
\mathbf{E}\varphi_{2,t} = \sum_{\forall \varphi_{\hat{1},t}} \min(k_2 q_1 + \cdots + (k_2 q_1 + (k_2 q_1 - k_1\varphi_{1,1})^+ - k_1\varphi_{1,2})^+ - \cdots - k_1\varphi_{1,t-1})^+, k_1\varphi_{1,t})\text{Prob}(\bar{\varphi}_{1,t} = \hat{\varphi}_{1,t})
$$

$$
= \mathbf{E}\varphi_{2,1} + (\mathbf{E}\varphi_{2,2} - \mathbf{E}\varphi_{2,1}) + (\mathbf{E}\varphi_{2,3} - \mathbf{E}\varphi_{2,2}) + \cdots + (\mathbf{E}\varphi_{2,t} - \mathbf{E}\varphi_{2,t-1})
$$

$$
= k_2 q_1 \left(\frac{1}{2} + \left[\frac{1}{2^2} + \frac{1}{2^5} + \cdots + \frac{1}{2^{t-1}}\frac{3}{2t-3}\left(\frac{t-3}{\frac{2t-6}{3}}\right)\right] + \left[\frac{1}{2^3} + \frac{2}{2^6} + \cdots + \frac{1}{2^t}\frac{3}{t}\left(\frac{t-2}{\frac{2t-3}{3}}\right)\right] \right)
$$

When $t \to \infty$

$$
\mathbf{E}\varphi_w = k_2 q_1 \left[\frac{1}{2} + \sum_{\substack{j=1 \\ t=3j}}^{\infty} \frac{1}{2^{t-1}}\frac{3}{2t-3}\left(\frac{t-3}{\frac{2t-6}{3}}\right) + \sum_{\substack{j=1 \\ t=3j}}^{\infty} \frac{1}{2^t}\frac{3}{t}\left(\frac{t-2}{\frac{2t-3}{3}}\right) \right]
$$

$$
= k_2 q_1 \left[\frac{1}{2} + \sum_{i=1}^{\infty} \frac{1}{2^{3i-1}}\frac{1}{2i-1}\left(\frac{3i-3}{2i-2}\right) + \sum_{i=1}^{\infty} \frac{1}{2^{3i}}\frac{1}{i}\left(\frac{3i-2}{2i-1}\right) \right]
$$

$$
= k_2 q_1 \left(\frac{1}{2} + \sqrt{\frac{2}{3}}\sin\left[\frac{1}{3}\arcsin\left(\frac{3}{4}\sqrt{\frac{3}{2}}\right)\right] + \frac{2}{3} - \frac{2}{3}\cos\left[\frac{2}{3}\arcsin\left(\frac{3}{4}\sqrt{\frac{3}{2}}\right)\right] \right)
$$

$$
= k_2 q_1
$$

Note that, in the long run, the wholesaler receives all his money, since payables are never released.

The second moment for $t = 1, 2, \ldots$ is

$$
\mathbf{E}\varphi_{2,1}^2 = \mathbf{E}\min(k_2 q_1, k_1\varphi_{1,1})^2 = k_2^2 q_1^2 \frac{1}{2}
$$

$$
\mathbf{E}\varphi_{2,2}^2 = \frac{1}{2^2}[(2k_2 q_1)^2 + k_2 q_1^2] = k_2^2 q_1^2 \left(\frac{1}{2} + \frac{3}{4}\right)
$$

$$
\ldots
$$

Following the same approach as with the first moment we obtain

$$
\mathbf{E}\varphi_w^2 = k_2^2 q_1^2 \left[\frac{1}{2} + 3\left(\frac{1}{4} + \frac{1}{32} + \frac{3}{256} + \ldots\right) + 5\left(\frac{1}{8} + \frac{2}{64} + \ldots\right) \right]
$$

$$
= k_2^2 q_1^2 \left[\frac{1}{2} + 3\sum_{i=1}^{\infty} \frac{1}{2^{3i-1}}\frac{1}{2i-1}\left(\frac{3i-3}{2i-2}\right) + 5\sum_{i=1}^{\infty} \frac{1}{2^{3i}}\frac{1}{i}\left(\frac{3i-2}{2i-1}\right) \right]
$$

$$
= k_2^2 q_1^2 \left[\frac{1}{2} + (2^2 - 1^2)S_{3,1} + (3^2 - 2^2)S_{3,2} \right] = k_2^2 q_1^2 (2 + 2S_{3,2})
$$

where $S_{3,1} \doteq \sum_{i=1}^{\infty} \frac{1}{2^{3i-1}} \frac{1}{2i-1} \binom{3i-3}{2i-2}$ and $S_{3,2} \doteq \sum_{i=1}^{\infty} \frac{1}{2^{3i}} \frac{1}{i} \binom{3i-2}{2i-1}$.

Given the expression of the two first moments, the calculation of the variance easily follows as

$$\mathbf{V}\varphi_2 = \mathbf{E}^2\varphi_2 - \mathbf{E}\varphi_2^2 = k_2^2 q_1^2 (1 + 2S_{3,2})$$

And the coefficient of variation is

$$\mathbf{CV}\varphi_w = \frac{\sqrt{\mathbf{V}\varphi_w}}{\mathbf{E}\varphi_w} = \sqrt{1 + 2S_{3,2}}$$

Since $CV\varphi_1 = 1$, the result for $c = 3$ follows.

For $c = 4$, given the relationship between the first two central moments, it can be shown that

$$\mathbf{E}\varphi_2^2 = k_2^2 q_1^2 \left[\frac{1}{2} + (2^2 - 1^2)S_{4,1} + (3^2 - 2^2)S_{4,2} + (4^2 - 3^2)S_{4,3} \right]$$

$$= k_2^2 q_1^2 (2 + 2S_{4,2} + 4S_{4,3})$$

with $S_{4,1} + S_{4,2} + S_{4,3} = 1/2$, and $S_{4,j} > 0$, $j = 1, 2, 3$.

In general, for any $c > 2$ integer, it can be shown that

$$\mathbf{E}\varphi_2^2 = k_2^2 q_1^2 \left[\frac{1}{2} + (2^2 - 1^2)S_{c,1} + (3^2 - 2^2)S_{c,2} + (c^2 - (c-1)^2)S_{c,c-1} \right]$$

$$= k_2^2 q_1^2 (2 + 2S_{c,2} + 4S_{c,3} + \cdots + 2(c-2)S_{c,c-1})$$

with $S_{c,1} + S_{c,2} + \cdots + S_{c,c-1} = 1/2$, and $S_{c,j} > 0 \ \forall j = 1, 2, \ldots$

Therefore

$$\mathbf{CV}\varphi_2 = \sqrt{1 + 2\sum_{j=2}^{c-1}(j-1)S_{c,j}} > 1$$

and the result follows for any $c > 3$ integer. $\quad \square$

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